7.1 Integral As Net Change

EXAMPLE 1 Interpreting a Velocity Function

Figure 7.1 shows the velocity

$$\frac{ds}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2} \qquad \frac{\text{cm}}{\text{sec}}$$

of a particle moving along a horizontal s-axis for $0 \leq t \leq 5$. Describe the motion.

Solve Graphically

EXAMPLE 2 Finding Position from Displacement

Suppose the initial position of the particle in Example 1 is $s(0) = 9$. What is the particle's position at **(a)** $t = 1$ sec? **(b)** $t = 5$ sec?

EXAMPLE 3 Calculating Total Distance Traveled

Find the *total distance traveled* by the particle in Example 1.

Strategy for Modeling with Integrals

1. *Approximate what you want to find as a Riemann sum* of values of a continuous function multiplied by interval lengths. If $f(x)$ is the function and $[a, b]$ the interval, and you partition the interval into subintervals of length Δx, the approximating sums will have the form $\sum f(c_k)\,\Delta x$ with c_k a point in the kth subinterval.
2. *Write a definite integral*, here $\int_a^b f(x)\,dx$, to express the limit of these sums as the norms of the partitions go to zero.
3. *Evaluate the integral* numerically or with an antiderivative.

EXAMPLE 4 Modeling the Effects of Acceleration

A car moving with initial velocity of 5 mph accelerates at the rate of $a(t) = 2.4t$ mph per second for 8 seconds.

(a) How fast is the car going when the 8 seconds are up?

(b) How far did the car travel during those 8 seconds?

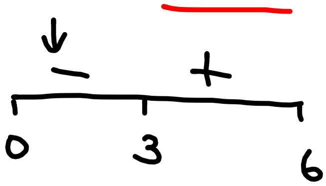
$$x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4$$

$$0 \le t \le 6$$

$$v(t) = t^2 - 2t - 3$$

$$0 = (t-3)(t+1)$$

$$\underline{t = 3} \qquad t = -1 \text{ not in the interval}$$



$$\left| \int_0^3 t^2 - 2t - 3\,dt \right| + \int_3^6 t^2 - 2t - 3\,dt$$

$$\left| \frac{1}{3}t^3 - t^2 - 3t \right| \Bigg|_0^3 + \frac{1}{3}t^3 - t^2 - 3t \Bigg|_3^6$$

$$\left| \left(\frac{27}{3} - 9 - 9\right) - (0) \right| + \left(\frac{216}{3} - 36 - 18\right) - \left(\frac{27}{3} - 9 - 9\right)$$

$$9 + 27 = \boxed{36}$$

EXAMPLE 6 Finding Gallons Pumped from Rate Data

A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for one hour as shown in Table 7.1. How many gallons were pumped during that hour?

Table 7.1 Pumping Rates

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

$$\int_0^{60} R(t)\,dt \approx$$ (where R(t) represents the rate function)

$$\frac{h}{2}\left(y_0 + 2y_1 + 2y_2 \ldots\ldots + 2y_{11} + y_{12}\right)$$

$$\approx 3582.5 \text{ gallons}$$

Work

In everyday life, *work* means an activity that requires muscular or mental effort. In science, the term refers specifically to a force acting on a body and the body's subsequent displacement. When a body moves a distance d along a straight line as a result of the action of a force of constant magnitude F in the direction of motion, the **work** done by the force is

$$W = Fd.$$

The equation $W = Fd$ is the **constant-force formula** for work.

The units of work are force $\times$ distance. In the metric system, the unit is the newton-meter, which, for historical reasons, is called a joule (see margin note). In the U.S. customary system, the most common unit of work is the **foot-pound.**

Hooke's Law for springs says that the force it takes to stretch or compress a spring x units from its natural (unstressed) length is a constant times x. In symbols,

$$F = kx,$$

where k, measured in force units per unit length, is a characteristic of the spring called the **force constant.**

EXAMPLE 7 A Bit of Work

It takes a force of 10 N to stretch a spring 2 m beyond its natural length. How much work is done in stretching the spring 4 m from its natural length?

$F = kx$

$10 = k(2)$

$k = 5$

$F(x) = 5x$

$W = Fd$

$\int_0^4 F(x)\,dx$

$\int_0^4 5x\,dx$

$\frac{5}{2}x^2 \Big|_0^4$

$\frac{80}{2} - 0 = 40\text{Nm}$ (J)